IMAGE RECOGNITION BY MULTIDIMENSIONAL INTERVALS

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ABSTRACT

In this paper new algorithm of interval images recognition is suggested. This algorithm gives accuracy solution of considered problem but demands not linear but square complexity by a number of objects. Main motive of such construction is to analyze practically interesting case when there is preliminary silence before predicted events.

1. PRELIMINARIES

In [1] the algorithm of interval images recognition is described. In a case of a single index which characterizes objects of first and second classes a minimal interval contained objects of the first class was constructed. Then an object with an index contained in this interval is considered as the first class object. In a case when each object is characterized by several indexes a one dimensional interval in a recognition procedure is replaced by a multidimensional interval constructed as a direct product of one dimensional intervals. An advantage of the interval images recognition before known algorithms is a linear (by a number of all objects and by a number of all indexes) calculation complexity. This algorithm is successfully used in manifold problems of medical geography and ecology, meteorology and fishing [2] – [9].

The algorithm is sufficiently satisfactory when a number of all objects in a sample is about 20-30 and a number of indexes is larger than 3. But in a problem arisen in a mining an emergence of a rock pressure cannot be predicted using a single interval. It means that there are first class objects which have predecessors and there are first class objects which have not predecessors. In this situation the single interval cannot characterize all first class objects because it goes past first class objects which may be described by a preliminary silence.

In this paper the method of interval recognition is developed in a direction of a consideration of such situation. It is based on a construction of few nonintersecting intervals which contain points characterized different first class objects. So first class objects are divided into some subclasses and their recognitions are realized separately. This algorithm is more complicated and has not linear but square complexity by a number of all objects.

2. MAIN RESULTS

Assume that first class objects are characterized by the set \( B = \{b_j, \ 1 \leq j \leq m\} \) and second class objects are characterized by the set \( A = \{a_i, \ 1 \leq i \leq n\} \), \(-\infty \in A, \infty \in A\), of real numbers. Suppose that \( m \) is much smaller than \( n \). For real numbers \( c, d, c \leq d \), define the interval \((c, d)\) by the
condition \( (c,d) = \{f : c < f < d\} \) if \( c < d \). If \( c = d \) then the interval \( (c,d) \) consists of the single point \( c = d \). Construct the following rule of a recognition of the object \( b \in B \). For each number \( b \in B \) contrast two numbers:

\[
k(b) = \max\{a \in A : a \leq b\}, \quad r(b) = \min\{a \in A : a \geq b\}.
\]

As a result we construct for each number \( b \in B \) the interval \((k(b), r(b))\).

**Theorem 1.** If \( b_i, b_j \in B \), then the intervals \((k(b_i), r(b_i))\), \((k(b_j), r(b_j))\) coincide or not intersect.

**Proof.** Assume that between the points \( b_i, b_j \) there are not points of the set \( A \). Then by a construction the intervals \((k(b_i), r(b_i))\), \((k(b_j), r(b_j))\) coincide. Vice versa if between the points \( b_i, b_j \) there are points of the set \( A \) then by a construction the intervals \((k(b_i), r(b_i))\), \((k(b_j), r(b_j))\) not intersect. So the points of the set \( B \) are divided into classes of an equivalence by their belonging to coincide intervals.

Suppose now that the set \( A \) consists of \( n \) objects and its each object \( i \) is characterized by \( l \)-dimensional vector \( a_i = [a_{i1}, \ldots, a_{il}] \). Analogously assume that the set \( B \) consists of \( m \) objects and its each object \( j \) is characterized by \( l \)-dimensional vector \( b_j = [b_{j1}, \ldots, b_{jl}] \). Define the interval \([k(b^j_i), r(b^j_i)]\) by the equality

\[
k(b^j_i) = \max\{a^j_i : a^j_i \leq b^j_i, \ 1 \leq i \leq n\}, \quad r(b^j_i) = \min\{a^j_i : a^j_i \geq b^j_i, \ 1 \leq i \leq n\}.
\]

Using these intervals construct \( l \)-dimensional interval which is its direct product.

\[
\otimes_{i=1}^l [k(b^j_i), r(b^j_i)].
\]

**Theorem 2.** If \( 1 \leq i \neq j \leq m \), then \( l \)-dimensional intervals \( \otimes_{i=1}^l [k(b^j_i), r(b^j_i)] \) coincide or not intersect.

**Proof.** Indeed, by a construction for any \( t, 1 \leq t \leq l \), one dimensional intervals \([k(b^j_i), r(b^j_i)]\), \([k(b^j_j), r(b^j_j)]\) coincide or not intersect. If these one dimensional intervals for all \( 1 \leq t \leq l \) coincide then their direct products \( \otimes_{i=1}^l [k(b^j_i), r(b^j_i)] \), \( \otimes_{i=1}^l [k(b^j_j), r(b^j_j)] \) coincide also. In opposite case there is \( t \) so that appropriate intervals not intersect and consequently their direct products not intersect also. Consequendty vectors from the set \( B \) are divided on subsets (equivalence classes) by their belonging to coincide \( l \)-dimensional intervals.

Suppose that \((l+1)\)-dimensional vectors arrive in recognition system. The first component equals zero if this vector belongs to the set \( A \) and equals one if this vector belongs to the set \( B \). Assume that on the step 0 two \((l+1)\)-dimensional vectors \((0, +\infty, \ldots, +\infty)\), \((0, -\infty, \ldots, -\infty)\) are introduced into the system. Further on the step \( n > 0 \) single vector \((\alpha_n, c_{n1}, \ldots, c_{nl})\) arrives. Denote \( n_0 \) the first vector for which \( \alpha_n = 1 \). Then the first multidimensional interval containing the vector \((c_{n1}, \ldots, c_{nl})\) is constructed.

Further assume that on the step \( n > n_0 \) the vector \((\alpha_n, c_{n1}, \ldots, c_{nl})\) arrives in recognition system. Suppose that \( \alpha_n = 0 \). Then if the vector \((c_{n1}, \ldots, c_{nl})\) does not belong to constructed intervals then the system of these intervals conserves. If \((c_{n1}, \ldots, c_{nl})\) belongs to one of constructed intervals then this interval is divided onto subintervals by described rule.

Assume that \( \alpha_n = 1 \) then if the vector \((c_{n1}, \ldots, c_{nl})\) does not belong to constructed intervals then new interval containing this vector is constructed. If \((c_{n1}, \ldots, c_{nl})\) belongs to one of constructed intervals then the system of intervals does not change.
REFERENCES